The formation of students’ mathematical competence through
competence-oriented tasks

La formación de la capacidad matemática de los estudiantes a través de
tareas orientadas a la competencia.

Natalya Vladimirovna Terekhova,1a Elena Aleksandrovna Zubova2

Industrial University of Tyumen, Tyumen, Russia1,2

ORCID ID: https://orcid.org/0000-0003-4570-62611
ORCID ID: https://orcid.org/0000-0001-5666-26342

Recibido: 18 de febrero de 2021 Aceptado: 09 de julio de 2021

Abstract
The aim of the article is to reveal the possibilities of the content of linear algebra teaching in the
formation of mathematical competence of future mechanical engineers. The implementation of the
experimental research involved the use of qualitative and quantitative methods: observations,
conversations with teachers of mathematical disciplines, testing, and mathematical processing of
research results. Methodological support of the course “Linear Algebra” was prepared, containing a
system of competence-oriented tasks. The study was conducted at the Industrial University of
Tyumen (Russia). Experimental (86 students) and control (87 students) groups were formed from
first and second-year students. In the control groups, training took place according to the traditional
method. Statistical analysis of the results obtained when comparing two empirical distributions was
carried out using the Pearson $\chi^2$ test. As a result of the study, the features of the academic discipline
“Linear Algebra” in the formation of mathematical competencies of students of technical universities
were identified. An approach to the formulation of a competence-oriented problem was developed.
The classification of tasks was carried out by types (algorithmic, research, computational,
instrumental, modeling) and levels (reproduction, establishment of connections, reasoning). The
results of the scientific and experimental work indicate an increase in the level of mathematics
competence of future mechanical engineers and fully confirm the effectiveness of the proposed
method of teaching linear algebra.

Keywords: Mathematics, competencies, education, algebra, formal science.

Resumen
El objetivo del presente artículo es revelar las posibilidades del contenido de la enseñanza del álgebra
lineal en la formación de la competencia matemática de los futuros ingenieros mecánicos. La
implementación de la investigación experimental involucró el uso de métodos cualitativos y
cuantitativos: observaciones, conversaciones con profesores de disciplinas matemáticas, pruebas y
procesamiento matemático de los resultados de la investigación. Se preparó el soporte metodológico
del curso “Álgebra lineal”, que contiene un sistema de tareas orientadas a competencias. El estudio
se realizó en la Universidad Industrial de Tyumen (Rusia). Se formaron grupos experimentales (86
alumnos) y de control (87 alumnos) a partir de estudiantes del primer y segundo año. En los grupos de control, el entrenamiento se llevó a cabo según el método tradicional. El análisis estadístico de los resultados obtenidos al comparar dos distribuciones empíricas se realizó mediante la prueba de la $\chi^2$ de Pearson. Como resultado del estudio, se identificaron las características de la disciplina académica “Álgebra lineal” en la formación de competencias matemáticas de estudiantes de universidades técnicas. Se desarrolló un enfoque para la formulación de un problema orientado a las competencias. La clasificación de tareas se realizó por tipos (algorítmica, de investigación, computacional, instrumental, modelado) y niveles (reproducción, establecimiento de conexiones, razonamiento). Los resultados del trabajo científico y experimental indican un aumento en el nivel de competencia matemática de los futuros ingenieros mecánicos y confirman completamente la efectividad del método propuesto para enseñar álgebra lineal.

**Palabras clave:** Matemáticas, competencias, educación, álgebra, ciencias formales.

**Introduction**

The implementation of the competence-oriented approach requires updating subject curricula with regard to the acquisition of key and subject-specific competencies. This is one of the main objectives of the modernization of the education system (Yarullin, Bushmeleva & Tsyarkin, 2015). In this case, the research problem lies in the need for a comprehensive analysis of the competency-based approach to teaching future mechanical engineers the fundamental and natural-scientific disciplines. This is confirmed by the existence of a contradiction between the capabilities of linear algebra in shaping mathematical competencies of future mechanical engineers (Alpers, 2010; Bamforth et al., 2007) and the lack of a developed competence-oriented teaching methodology (Lavrinenko & Mikhno, 2017; Novak, 2017). On the one hand, training should contribute to the fundamental knowledge of students (Stefanova, Krutova & Valisheva, 2011). On the other hand, it should be aimed at the formation of the mathematical competence of the future mechanical engineer (Mason, 2020; Berliner, 2015). Thus, the solution to the problem of teaching students of technical specialties linear algebra is relevant.

Teaching linear algebra to future mechanical engineers based on the competence-oriented approach requires rethinking the learning content (Yarullin, Bushmeleva & Tsyarkin, 2015; Niss, 2003). Formation of mathematical competencies in the process of mastering the content of the discipline “Linear Algebra” implies the appearance of new types of tasks in the content, namely competence-oriented tasks. We propose to enrich the traditional content of the course “Linear Algebra” for future mechanical engineers with competence-oriented tasks.

Tasks aimed at applying the acquired knowledge in situations similar to the future professional activity have an important place in the professional training of a specialist (Bondarenko et al., 2020; Tereshchenko et al., 2020; Efremenko et al., 2020, Lutsan et al., 2020). Competence-oriented tasks have the greatest potential in this direction allowing making tasks simulating actual problems of their
future professional activity the subject of learning activities of future mechanical engineers. Among such tasks, there can be research-type tasks as well.

**Literature review**

The problem of the formation of mathematical knowledge and skills of students has been actively discussed by researchers on the pages of specialized international publications over the past five years. The main issues are the possibility of using innovative technologies in the educational process of students in the study of mathematical disciplines (Szabo et al., 2020; Tovarnichenko, Stepkina, 2015), providing fundamental mathematical training of future engineering specialists (Cardella, 2008; Baturina & Khasanova, 2020), and conditions providing an increase in the level of mathematical training of university students (Alpers, 2010; Lavrinenko & Mikhno, 2017). Some specialists have tried to solve the problems of developing the mathematical culture of students of different specialties (Bamforth et al., 2007) and attracting students to the study of mathematical disciplines (Novak, 2017; Anufrieva et al., 2020; Akhyadov et al., 2020).

The analysis of scientific research and practice of mechanical engineers' training shows that the formation of students' professional competencies is mainly concentrated in their psychological-pedagogical, methodological, and professional scientific and subject-specific training. Therefore, it is necessary to pay attention to the achievements of scholars (Yarullin & Bushmeleva, & Tsyrkun, 2015; Niss, 2003; Henderson & Broadbridge, 2009), which refer to individual academic disciplines, among which linear algebra deserves special attention.

Teaching linear algebra requires considerable effort from students and contributes to the development of logical thinking, attention, and memory, i.e. forms their learning competencies (Yarullin et al., 2015). The application of numeracy skills, algorithms of actions, and information and communication technologies in the process of solving linear algebra problems contributes to the formation and development of professional competencies (Niss, 2003). The knowledge of linear algebra forms the basis for acquiring practical skills in solving tasks of different complexity and helps to develop individual qualities in mental processes: logic, consistency, analyticity, etc. (Henderson & Broadbridge, 2009). Researchers distinguish between professionally-oriented and competence-oriented tasks (Table 1).
Table 1

*Characteristics of competence-oriented tasks*

<table>
<thead>
<tr>
<th>Source</th>
<th>A characteristic feature of a group of tasks</th>
<th>Description of the tasks of the group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lein et al., 2020</td>
<td>Problem-based nature</td>
<td>the problem statement includes an educational or training-professional problem</td>
</tr>
<tr>
<td>Mason, 2020</td>
<td>Irregularity in the formulation of the problem</td>
<td>may have insufficient or excess data and suggest alternative solutions</td>
</tr>
<tr>
<td>Rosli, Goldsby, Capraro, 2013</td>
<td>Educational orientation</td>
<td>the aim of solving such tasks is to acquire new knowledge, skills, and activities that are cognitively, professionally, personally, and in other ways meaningful to the student</td>
</tr>
<tr>
<td>Alcock et al., 2016</td>
<td>Activity-oriented nature</td>
<td>the student operates with a body of knowledge on one or more topics, undertaking a variety of activities to solve such a task each task is followed by tasks to help one master the techniques of working with the proposed content</td>
</tr>
<tr>
<td>Stefanova, Krutova, Valisheva, 2011</td>
<td>Existence of subtasks</td>
<td>the content of the tasks should include both self-assessment and analysis of the process of completing the task; formulating one's attitude towards the obtained result</td>
</tr>
</tbody>
</table>

Thus, we believe it expedient to consider competence-oriented tasks as specially designed tasks aimed at developing a dynamic combination of knowledge, abilities, and practical skills, ways of thinking, and professional and outlook qualities that allow successfully carrying out further educational and professional activities. The hypothesis of the research: application of competence-oriented tasks in the process of teaching linear algebra will lead to improvement of mathematical competence of future mechanical engineers.

**Methodology**

**Study design**

To prove the formulated hypothesis, an experimental study was carried out on the formation of mathematical competencies of future mechanical engineers through competence-oriented problems of linear algebra. The purpose of the experimental study is to determine the level of mathematical competence of future mechanical engineers. To implement the main tasks of the experimental research, qualitative and quantitative research methods were used: observations, conversations with teachers of mathematical disciplines (in particular linear algebra) in technical universities, surveys of students and teachers in technical universities, student testing, control papers, mathematical processing of research results (Yarullin, Bushmeleva, & Tsyrkun, 2015).

The experimental and research work at this stage allowed us to identify ways to improve the efficiency of linear algebra training at a technical university, the acquisition of basic mathematical competencies by future mechanical engineers; also, to outline psychological and pedagogical
peculiarities of forming mathematical competencies of future mechanical engineers at a technical university; to outline directions and tasks for the next stages of the pedagogical experiment; in addition, to confirm the relevance and necessity of developing and implementing competence-oriented linear algebra tasks in the process of mathematical training of students (Mason, 2020; Berliner, 2015).

**Research instruments and procedure**

The study was conducted at the Industrial University of Tyumen (Russia) in 2019. At the exploratory stage of the experiment, methodological support for the course “Linear Algebra” (approved by the educational and methodological university council), was prepared. The system of competence-oriented tasks, which are aimed at forming all components of mathematical competence of future mechanical engineers in the process of training in linear algebra, is provided in the content of the course (Szabo et al., 2020; Tovarnichenko & Stepkina, 2015). An electronic training course “Linear Algebra and Analytic Geometry” in the distance learning support system “Moodle” is proposed for the organization of students' self-study.

The forming phase was aimed at approbation, correction, and implementation of the competence-oriented methodology of teaching linear algebra to future mechanical engineers in the real learning process. The experimental (EG — 86 students) and control (CG — 87 students) groups were formed of first- and second-year students. The experimental groups were taught according to our competence-oriented method of teaching linear algebra, which provided for the introduction of competence-oriented tasks into the content of linear algebra teaching. The control groups were taught according to the traditional teaching method. At the beginning of the experiment, the law of normal distribution was confirmed; the EG and CG had no statistically significant differences in the distribution of students' scores.

**Statistical analysis**

The level of solving competence-oriented problems determined the formation of the mathematical competence of the future mechanical engineer. The effectiveness of supplementing the learning content of linear algebra with competence-oriented tasks was tested by determining the degree of formation of axiological, gnoseological, and praxeological components of mathematical competencies during the study of linear algebra. The weights for these components in the total formation of mathematical competencies (0.2, 0.35, and 0.45, respectively) were determined.

The level of formation of the axiological component demonstrated the awareness regarding value orientations, motives, and interests aimed at algebraic training, future mechanical engineer's need for creativity, readiness to independently formulate goals of professional and creative activity.
and achieve them, aspiration for volitional tension, and attitude for improvement of their professional experience. The gnoseological component reflects the quality of theoretical and practical knowledge which is formed in the process of training in linear algebra and their use in problem-solving. The level of formation of the praxeological component is distinguished by the degree of comprehension of actions aimed at self-regulation, the ability to make decisions, consciously approach the solution of the task, communicative competence, and adequate self-esteem.

To diagnose the dynamics of the levels of mathematical competence formation, the results of the competence test were used, which was assessed on a 100-point system. The scores received by students were distributed according to the levels of mathematical competence formation in linear algebra as follows:

Grade A — 80-100 points — high level,
Grade B — 60-79 points — sufficient level,
Grade C — 50-75 points — average level,
Grades D, E, FX — 40-49 points, 30-39 points, 0-29 points, respectively — low level.

Statistical analysis of the experimental results obtained when comparing two empirical distributions according to the Pearson $\chi^2$ test was carried out using the SPSS statistical software package.

**Results**

The practical significance of the research lies in the development of a system of competence-oriented tasks in accordance with the content of the discipline “Linear Algebra.” The article presents a system of tasks on the topic “Matrices and determinants”. The positive statistically significant dynamics of the indicators of future mechanical engineers' competence level shows that in organizing the process of linear algebra training, it is necessary to consider such factors of the linear algebra training process as: providing the formation of general and professional competencies of future mechanical engineers (Table 2).
Table 2
Comparative distribution of students according to the levels of competence formation

<table>
<thead>
<tr>
<th>Formation level of mathematical competence</th>
<th>Number of students, % Before the formative experiment</th>
<th>Number of students, % After the formative experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EG (n=86)</td>
<td>CG (n=87)</td>
</tr>
<tr>
<td>High</td>
<td>15.9</td>
<td>17.3</td>
</tr>
<tr>
<td>Sufficient</td>
<td>43.0</td>
<td>42.5</td>
</tr>
<tr>
<td>Average</td>
<td>31.4</td>
<td>32.2</td>
</tr>
<tr>
<td>Low</td>
<td>8.1</td>
<td>8.0</td>
</tr>
</tbody>
</table>

The level of formation of mathematical competence

<table>
<thead>
<tr>
<th>Number of students, %</th>
<th>EG</th>
<th>CG</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>22.76</td>
<td>17.81</td>
</tr>
<tr>
<td>Sufficient</td>
<td>52.85</td>
<td>42.11</td>
</tr>
<tr>
<td>Average</td>
<td>19.92</td>
<td>30.77</td>
</tr>
<tr>
<td>Low</td>
<td>4.47</td>
<td>9.31</td>
</tr>
</tbody>
</table>

The histogram of the comparative distribution of students (in percentages) by the level of competence in linear algebra is shown in Figure 1.

![Figure 1](image)

Figure 1
The distribution of students in the experimental and control groups after the completion of the formative experiment (in %)

As a result of statistical analysis of the obtained results, comparing two empirical distributions using Pearson's χ² criterion, it is confirmed that the formation of students' competencies is effective due to structuring the content of the “Linear Algebra” discipline.
Discussion

Authors will examine in more detail the justification of methodological support for the course “Linear Algebra”, the content of which provides for a system of competence-oriented tasks. Depending on what kind of mathematical competencies will be formed while solving competence-oriented tasks, we grouped competence-oriented linear algebra tasks into the following types: algorithmic, research, computational, instrumental, and simulation (Xu et al., 2019). The mathematical competencies of a future mechanical engineer, which can be formed in linear algebra training, determine the following types of competence-oriented tasks in linear algebra course content: algorithmic, research, computational, instrumental, simulation (López, 2011).

By algorithmic competence-oriented tasks we understand the tasks which will contribute to the formation and development of algorithmic competencies. According to Alpers (2010), in linear algebra, we refer to tasks for application of algorithms of operations over matrices, finding matrix rank, calculating determinants of different orders, finding inverse matrix to the given one, constructing the orthogonal and normalized basis of space, investigating systems of linear equations for compatibility and definiteness and finding their solutions, reducing quadratic forms to canonical form, finding rank and basis of vector system, etc. One has to select and correctly apply algorithms, check or establish the correctness of an algorithm, and partially create one’s algorithms, since the study of Rensaa et al., 2020).

By research competence-oriented tasks we understand the tasks which will contribute to the formation and development of research competencies. In linear algebra, according to researchers Henderson and Broadbridge (2009), we are talking about tasks in which we need to investigate: systems of linear equations for compatibility and definiteness, systems of vectors for linear dependence and linear independence, several vectors for orthogonality, transformations for linearity checking, finding rational method for solving linear algebra and other disciplines problems, etc. We need to analyze, research, and find data independently and process various kinds of information.

By computational competence-oriented tasks we will mean the tasks which will contribute to the formation and development of computational competencies In linear algebra, according to researchers Alpers (2010), Niss (2003), Henderson and Broadbridge (2009), such tasks are to apply methods of operations over matrices, calculating matrix rank, solving systems of linear equations, reducing quadratic forms to canonical form, etc. while studying other disciplines and using mathematical methods in the process of solving applied problems.

By instrumental competence-oriented tasks we will mean those tasks which will contribute to the formation and development of instrumental competencies. In linear algebra, according to researchers Novak (2017); Tovarnichenko and Stepkina (2015), Henderson and Broadbridge (2009), these are tasks for composing procedures for solving systems of linear equations, calculating
determinants and inverse matrices, investigating linear dependence and independence of a vector system, etc. To solve them, ICT can be used for data research and analysis, as well as information processing.

By simulation competence-oriented tasks we understand those tasks which will contribute to the formation and development of simulation competencies. In linear algebra, according to Cardella (2008), these are tasks for the application of linear algebra methods and techniques to solve applied problems and investigate environmental processes and phenomena. They use mathematical modeling to build models of cybernetic and stochastic processes and algorithms for studying and building models of phenomena and processes, as can be seen in Novitsky (2020).

It is distinguished, by analogy with the PISA tests (Berliner, 2015), three levels of competence-oriented problem solving: 1) reproduction level, 2) establishing connections, 3) reasoning level. The highlighted levels are based on the level of students' mathematical background:

1. The first level (reproduction level) involves recognizing, reproducing, and applying features of concepts, methods, and algorithms of calculation. Students can use basic concepts to investigate and analyze typical and specifically formulated typical situations. They can solve one-step applied problems and identify simple algebraic relationships, a common system of notation. They can recognize and transform applied problem data, which can be written in tables, or represented graphically in the form of graphs, maps, and various scales (Hasibuan & Fauzi, 2020; Siregar, Asmin, & Fauzi, 2018).

2. The second level (establishing connections level) implies recognizing and investigating connections and using the acquired knowledge from different mathematical sections that are needed to solve a given problem. In this case, as in the study of Simanjuntak, Napitupulu and Manullang (2018), students can apply the learned knowledge in atypical, clearly articulated complex situations. They can make relationships, analyze, investigate, calculate, and solve multi-step application problems. Students can solve simple algebraic problems that involve constructing algebraic expressions, solving systems of linear algebraic equations, and calculating values of certain quantities, while applying learned formulas and algorithms. They can interpret information that is presented in tables, graphs, etc. (Rosli et al., 2013; Blomhøj, Jensen, 2003).

3. The third level (reasoning level) implies generalization of mathematical data, their analysis and synthesis, construction and proving or disproving simple mathematical hypotheses, statements, and mathematical reasoning. In the solution of tasks of the third level, one should independently identify and analyze the problem in the given situation, find a way to solve it by means of mathematical techniques and methods, and be able to build a mathematical model of the problem to be solved. Students can organize data on their own, systematize it, solve atypical, rather complicated problems, work with the data, and draw conclusions, as it can be seen in Bamforth et al., 2007; and of
Mason (2020). They can navigate in unknown situations, using the mathematical knowledge acquired and finding the dependencies between them, and make algebraic models for the problems to be investigated. They can translate data that is presented in a non-standard form from one form to another and vice versa. Solve a proposed non-standard problem using mathematical concepts and statements, and interpret the answer according to the problem posed in the task is possible for them as well (Beklemisheva et al., 2004; Proskuryakov, 2005; Efimov & Pospelov, 2001).

The level-based approach to constructing competence-oriented tasks in linear algebra contributes to both attracting the leading component of learning activity and improving it (Stefanova et al., 2011). To effectively master the content of linear algebra and form students' competencies, both traditional and computer-oriented learning tools have been tested while solving competence-oriented tasks (Bondarenko et al., 2020). The limitations of the study should include the future professional specialization of students (mechanical engineers), which affects the weight of the axiological, epistemological, and praxeological components of mathematical competencies in the general formation of mathematical competencies, the preparation of methodological support within the framework of only one training course (“Linear Algebra”) with a limited number of competence-oriented tasks, as well as a limited sample (173 people).

**Conclusion**

The results of the research and experimental work show that the level of mathematical competence of future mechanical engineers increased and it fully confirms the effectiveness of the proposed methodology of linear algebra training. Thus, the hypothesis that the application of competence-oriented tasks in linear algebra training will improve the mathematical competence of future mechanical engineers has been confirmed.

However, the conducted research does not cover all the problems of improving the mathematical training of engineering students. There are promising studies focused on the development of integrative methods of studying mathematical disciplines for students of technical specialties.

**References**


Novitsky, N. (December 11, 2020). Mathematical modeling of hydraulic chains as cyber-physical objects [Rudenko International Conference on Methodological Problems in Reliability Study of Large Energy Systems]. E3S Web of Conferences, Kazan Russia. [https://doi.org/10.1051/e3sconf/202021601091](https://doi.org/10.1051/e3sconf/202021601091)


